TASKS TO SUPPORT PRESERVICE TEACHERS' UNDERSTANDING OF DECIMAL QUANTITIES

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In this article, we present some mathematical tasks designed to support pre-service teachers (PSTs) in obtaining a broader, more connected understanding of decimal numbers and quantities. We describe classroom scenarios from a content course in which PSTs experienced these cognitively demanding decimal tasks, enacted in ways that maintain high demand. We present evidence of shifts in quality of representations and explanations from pre- and post-test responses of PSTs to unique decimal comparison tasks. Prior to a unit on decimal fractions, several PSTs utilized incorrect decimal notation for amounts such as "0.16" for "16-tenths," relying on symbolic representations they may not fully understand. Written explanations of decimal comparisons commonly referred to procedural rules. After the decimals unit, responses to comparison tasks showed an increase in use of meaningful, connected representations and richer, more robust justifications.

Keywords: Teacher Education-Preservice, Rational Numbers, Mathematical Knowledge for Teaching

Teaching mathematics for understanding requires teachers to know the mathematics they teach with depth and flexibility (CBMS, 2001; CBMS, 2012; Ball, Hill, & Bass, 2005). However, studies have shown that elementary teachers in the U.S. appear to lack profound mathematical understanding (Ma, 1999). To support preservice teachers (PSTs) in experiencing mathematics as a subject that relies on reasoning and in learning mathematics with understanding, we have been developing and modifying curriculum materials to be used in content courses for PSTs.

Based on the recommendations of Stein and colleagues (2009), these curriculum materials feature cognitively demanding tasks, which require learners to solve problems in multiple ways, to make connections among representations and solution methods, and to explain their thinking. Critical in the use of such tasks to develop understanding in children is the teacher's ability to maintain a high level of cognitive demand during classroom implementation. Building on students' prior knowledge, allowing sufficient time to explore, and pressing students to justify their reasoning are all factors associated with the maintenance of high-cognitive demand of the tasks in classrooms of children. We wondered: Do these same features of mathematical tasks and their enactment in cognitively demanding ways also support a diverse group of adult students (PSTs) to learn mathematics in deeper, more connected ways? Can we measure PST improvement in their abilities to represent mathematical ideas in multiple ways, to make connections among those representations, and to use them to help justify why the mathematics works? These were questions we set out to study.

In this report, we present some of the tasks designed to help our PSTs develop meanings and representations of decimal fractions, content shown to be challenging to both children and their adult teachers (D'Ambrosio & Kastberg, 2012; Martinie & Bay-Williams, 2003; Putt, 1995; Stacey et. al., 2001). In particular, we examine some of the features of decimal tasks and their enactment that seem to play a role in broadening PSTs' conceptions and representations of

decimals. To measure shifts in quality of representations, depth of reasoning, and validity of justifications, we analyzed PST responses from a pair of items on pre- and post-tests that challenged PST reasoning about decimal quantities.

Theoretical Framework

An understanding of decimal numbers and quantities requires knowledge of both base-ten place value and fractions, two conceptual fields that are known to be difficult for learners (Fuson, 1990; Kamii, 1986; Luo et al., 2011; Post et. al, 1991; Ross, 1989; Siebert & Gaskin, 2006; Sowder et al., 1998; Thanheiser, 2009; Zazkis & Khoury, 1993). Limited understanding of base ten relationships in decimal notation and of connections between fractions and decimals leads to patterns of decimal comparison errors that have been well studied (Resnick, et al.,1989; Stacey et al., 2001; Steinle & Stacey, 2004). Because the Common Core State Standards for Mathematics introduce decimal fractions in grade 4 as, "Understand decimal notation for fractions, and compare decimal fractions" and build on this knowledge in later grades (CCSSI, 2010), elementary teachers need to possess a deep and well-connected understanding of the base-ten and fractional nature of decimal numbers. Due to the diversity present in nearly every classroom, teachers should be able to use a variety of representations and approaches to support this understanding.

To address shallow and incomplete understandings, several researchers have suggested activities to help with making connections among multiple representations (Suh et al., 2008). Ten-by-ten grids, that embody the base-ten structure of decimals, have been proposed to help learners visualize the order of magnitude represented by place value positions (D'Ambrosio & Kastberg, 2012; Martinie & Bay-Williams, 2003) and Cramer and colleagues (2009) emphasized making connections between and among representations (symbolic, pictorial, and verbal). In an earlier report, (Rathouz & Cengiz, 2012), we found that introducing PSTs to such diagrammatic representations as well as appropriate verbal re-namings (e.g. "two-tenths" rather than "point two" for 0.2), helped them to make sense of the relative sizes of decimal quantities.

Presenting representations imbedded in tasks that require a high level of cognitive demand has been shown to further support children's learning. Stein and her colleagues (2009) found that asking learners to solve problems in more than one way, to make connections among solution methods, and to explain their thinking, all contribute to the cognitive demand of the task. However, maintaining a high level of demand during classroom instruction was found to be just as important in children's learning. What does this mean for our courses where adult PSTs are trying to make sense of prior mathematical knowledge that they themselves will need to teach? In this study we investigate whether enactment of mathematical tasks in ways that have been shown to support children's learning also foster deep, connected understanding in adult learners. We conjecture that additional task features, such as reflecting on and reasoning about others' mathematical thinking, seem to contribute further to the demand while introducing PSTs to work they will be doing to support learning in diverse populations of their future students.

Methodology

The data for this study were gathered from the first of three mathematics content courses for preservice elementary and middle school teachers. The subjects were 160 PSTs enrolled in this course during one of five semesters (Winter 2008 – Fall 2010). The authors were the instructors for the course sections during this time. The data include pre- and post-tests on PSTs' knowledge of decimals, PSTs' written work, and videotapes of lessons from the decimals unit.

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In this report, we present data on one item from pre- and post-tests, describe mathematical tasks implemented during the unit on decimals, and provide sample classroom discussion to give insight into how shifts in thinking about decimals might happen in such environments.

Pre- and Post-Test

The pre-test was administered before the decimals unit began and the post-test was given at the end of the unit. PSTs were given 15 minutes to complete the items and neither test was used in determining course grade. Unlike typical decimal comparison items where numbers are written in decimal form (Stacey et. al, 2001), this pair of corresponding items, listed in Table 1, required PSTs to compare two amounts that are written in numerals and words, necessitating the coordination of at least two representations. Further, PSTs were asked to provide explanations for their comparisons. Many used diagrams to justify their thinking.

Table 1. Items About Comparing Decimals.

Pre-test Item	Post-test Item
Which one is bigger: **16-tenths** or **134-hundredths**? Show all work and briefly explain how you decided which is larger.	Which one is bigger: **18-tenths** or **172-hundredths**? Show all work and briefly explain how you decided which is larger.

The analysis of PSTs' representations and written work on these decimal comparison items was an iterative process, briefly described here and in more detail in a prior report (Rathouz & Cengiz, 2012). First, we categorized the types of representations that were used: symbolic; verbal; everyday life context; and diagrams; and the correctness of the representations. Next, we determined strategies for **renaming the quantities**, including (a) finding a common denominator; (b) making a whole with some leftover; and (c) positioning the right-most digit of the numeral in the named place value position (i.e. for 134 hundredths, the "4" would be placed in the hundredths place); (d) using division. The strategies (including both correct and incorrect ones) for **comparing the quantities** were categorized as (a) using common denominators or same length decimals; (b) using a benchmark; (c) comparing matching place value positions; (d) comparing the denominators or place value names/positions (i.e., tenths are larger than hundredths); (e) comparing the numerators or digits; (f) shorter is larger (e.g., 0.6 is closer to the decimal point); (g) longer is larger; (h) erroneous place value comparison; and others.

PSTs' explanations for both renaming and comparing decimals were evaluated using the following levels: explicit and valid explanation (3); partial explanation (2); description of the process or rule without explaining why (1); no explanation, only computation or representation (0); and confusing/incorrect (-1). The structure of the item revealed PSTs' strategies of representing, renaming, and comparing decimal quantities. Thus, PST responses on these items were used as one measure of effectiveness of the decimal curriculum materials and their classroom implementation.

Decimal Tasks and Enactment

Lessons in the content courses are 100 minutes long and focus on two or three tasks during one session. These tasks are designed to provide PSTs with opportunities to explore and make connections among important mathematical ideas. PSTs typically work on the tasks first individually; then share their thinking in small groups. Finally there is a discussion where solutions are shared and synthesized as a whole group. The focus of in-class discussions is not only on how problems are solved, but, importantly, why the solutions make sense. The expectation of justifying solutions and mathematical claims challenges PSTs' thinking.

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The unit on decimals is designed to support PSTs' developing knowledge about meanings and representations of decimals (such as currency,10-by-10 grids, fraction notation and place-value language) and connections among these meanings and representations. Throughout the unit PSTs are encouraged to use both meanings and representations to support their justifications. See Table 2 for a selection of tasks from the decimals unit.

Table 2. Selected Decimals Tasks From Coursework

Task A: Renaming 0.1	Rename this number [0.1] in more than one way. You could use numbers, words, drawings, and/or contexts.
Task B: Representing fractions as decimals	Find four different ways to show 1/4 on a 10 x 10 grid by shading in that portion of the grid. Try to choose ways that allow you to clearly see the result is 1/4 without requiring that a person count every shaded box. Explain how your ways also show the decimal for ¼.
Task C: Analyzing students' re-namings	Several students were discussing different ways to name 245-hundredths. In evaluating the following students' claims, first create your own representation of 245-hundredths using 10 x 10 grids. Bradley said that 245-hundredths could be represented as $\frac{2450}{1000}$. Use both words and a diagram to validate Bradley's claim. Amira said that she could write 245-hundredths as 0.245. Use both words and a diagram to persuade Amira that these two expressions are <u>not</u> equivalent.

Results

Implementation of Decimals Tasks

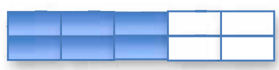
As a formative assessment of PSTs' conceptions of decimal fractions and notation at the beginning of the unit and to reinforce the course emphasis on making sense of numbers and operations, we pose the following for the whole class to consider: "Rename this number [0.1, written on the board, not verbally read] in more than one way. You can use numbers, words, drawings, and/or contexts." The task of finding ways to rename 0.1 [Table 2, Task A] provides opportunities to raise PSTs' awareness of their dependence on previously learned, but not understood, place-value rules and to make sense of the rules by pressing PSTs to think about the structure of our number system. Their challenge is to produce arguments that convince others that their renamings are equivalent to 0.1.

The PSTs begin by working independently to give individual time to think and to generate a greater variety of responses. They share briefly their renamings in small group discussions. This gives reluctant participants a chance to try out their ideas and to begin to challenge each other to explain how they know that their renaming is equivalent to 0.1. The small group discussions also provide the instructor with a window into the kinds of thinking present in the group as a whole. She can choose, from among the PSTs' renamings she sees and hears, those she wishes to pursue in the whole-group discussion. The sample discussion below exemplifies the kinds of reasoning we have heard from PSTs in our courses. Such typical dialogue is also provided in the lesson plans to aid in instructor preparation. All PSTs have been given pseudonyms.

Instructor: We have a lot of different ways you thought of to rename this number. Is everyone comfortable with all of these or are there ones you are questioning?

Katy: Well, at first I wrote ".6 + .4 = .10," but I think it's wrong now.

Instructor: What made you change your mind about that one, Katy?
Katy: I was thinking….that's "point six and point four equals point ten." Then Jason reminded me that point six is really six-tenths and that's already bigger than point one…I mean, one-tenth.
Instructor: Something we all need to practice is using language that is meaningful. Instead of using "point" language, how did it help to say "six-tenths?"
Amanda: It helped me when I wrote ".01 + .09 = .1" as my renaming. 'Cause I thought back to fractions like one-hundredth and nine-hundredths: [writes on board $\frac{1}{100}+\frac{9}{100}=\frac{10}{100}$]. And I think you can write that fraction like this ".1" as a decimal.
Instructor: What do others think about Amanda's justification? What else might someone say to help understand her way of renaming this number [0.1]?
Asma: You could think about it as, like, money. I wrote, "0.01 x 10 = 0.1," like one penny is a hundredth of a dollar and I've got ten pennies, so that's a dime. Amanda just did one penny and nine pennies.
Jason: I wanted to see if I could use division, so I wrote, ".6 ÷ 6 = .1" and drew this diagram.



Instructor: *Hold on a sec… don't tell us about the diagram yet, Jason. Can someone besides Jason figure out how his diagram might help to show his renaming equation?*
Samantha: *I guess it's showing a whole of something, maybe a pan of brownies. Then there is only point-six or six-tenths left of the pan and you are sharing that amount with six people, so everyone gets a tenth.*
Instructor: *That was very nice, Samantha, to connect Jason's equation back to a sharing meaning of division.*
Zeinab: *We were talking about that one [0.025 x 4] at our table….None of us felt comfortable with how to explain it.*
Nichole: Now, when I see something like that, I always multiply 25 times 4, which is 100 and then remember to move the decimal point three places for decimal places in 0.025 so that give you .100. But I guess that sounds kind of like a rule.

Even in this seemingly trivial initial task, many mathematical ideas are introduced, representations shared, and class norms re-established. For example, Katy acknowledges an incorrect idea first and shares thoughts on editing. Amanda makes connections to a course unit on fractions recently completed. Asma considers currency as a real-world use of decimal numbers and Jason reminds the class of area models and a meaning for the operation of division. Typically, the instructor will prompt the PSTs to comment on others' solutions and to use more helpful fraction language rather than "point" language. Zeinab's question about a number with three decimal places provides an opportunity for others to enter a discussion regarding the structure and language of the base-10 place value system. Nichole's computation shows that, at this early stage in the unit, some PSTs' explanations are based on rules. As students attempt to show decimals in the thousandths place, they require the discussion of very small pieces. This fact motivates the use of 10-by-10 grids and of more descriptive place-value language.

A second task [Table 2; Task B] continues to help connect familiar decimal numbers to their fraction equivalents using 10-by-10 grids as well as students' own representations. Representing ¼ several ways on the 10-by-10 grids, PSTs visualize 0.25 as both "2 tenths plus 5 hundredths"

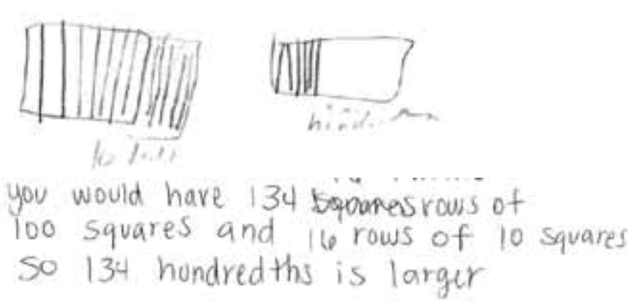
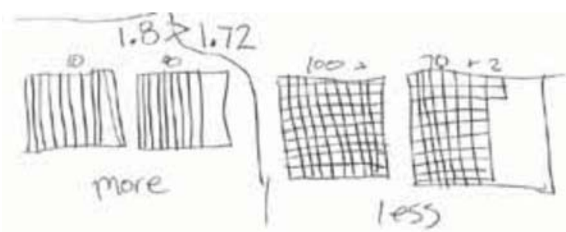
and "25 hundredths." As they move to producing 1/8 on the 10-by-10 grid, they run up against further renaming issues. Some use the relationship between ¼ and 1/8. They figure that 1/8 is half of 25 hundredths and arrive at 12 ½ hundredths as their equivalent. Others write the decimal as 0.125. It is then the work of the entire class to justify why 125 thousandths (0.125) is the same as 12 ½ hundredths. This is non-trivial work and involves using base-10 language and ideas. For example, one hundredth must be viewed as both split in half and split into ten equal thousandths with five of those thousandths shaded.

A third task involves renaming a decimal quantity and evaluating other students' reasoning about equivalence [Table 2, Task C]. The PSTs work on the task as homework and then share ideas during a whole-class discussion. As they evaluate Bradley's and Amira's reasoning, PSTs confront their own incorrect ideas about renaming decimal quantities, use representations to clarify their thinking, and work on arguments to convince others that quantities are equivalent. For example, Amira's solution provides a typical error similar to one PSTs made on the pre-test. Evaluating the correctness of other students' renamings requires the use of 10-by-10 grids or other diagrams. The introduction of fraction notation and verbal representations (such as 24-tenths + 5-hundredths; a third renaming suggested) provide fodder for discussion of connections to the base-ten structure of decimal numbers.

Shifts in Representations

To document the types of learning that PSTs experience during tasks in the decimals unit described above, brief pre- and post-assessments were administered containing items that required PSTs to represent, compare, and order decimal quantities. The most dramatic changes we noted in PSTs' representations were shifts from incorrect decimal notation [0.16 for 16-tenths and 0.134 for 134-hundredths] on the pre-test to correct decimal notation on the post-test. Additionally, PSTs began to utilize 10 x 10 grids to represent the quantities (see Table 2) and to successfully compare those quantities (Rathouz & Cengiz, 2012).

Table 2. Sample PST Responses for Using 10-by-10 Grids

An incorrect response from the pre-test	A correct response from the post-test
16-tenths vs. 134-hundredths  	18-tenths vs. 172-hundredths 

Strategies and Explanations

The explanations for and reasoning about why one quantity was larger than the other also shifted in two critical ways from the pre- to the post-test. The first change we documented was a decrease in the use of incorrect or incomplete strategies. For example, on the pre-test, eighteen PSTs (11%) held the misconception that numbers with fewer digits past the decimal place were always smaller (the so-called "shorter is larger" misconception). Such PSTs claimed, "16 tenths is bigger because tenths is closer to the decimal, meaning it's closer to being a whole number."

On the post-test, only five (3%) of the PSTs used this incorrect idea. Interestingly, these five PSTs had used other incorrect strategies on the pre-test to explain their comparison.

Another 11% of PSTs on the pre-test held a related, incomplete conception that any number of tenths is larger than any number of hundredths: "When you have 16 tenths, it's bigger because each piece is bigger. With hundredths, you are basically chopping the tenths up into hundredths." Again, observation of this strategy decreased on the post-test to 5% (8 PSTs), the majority of these PSTs having used other erroneous strategies on the pre-test. Twenty-two PSTs (nearly 14% on the pre-test) were unaware of the distinction in language between "tens" and "tenths" and, therefore, compared the quantities incorrectly. This error was completely absent on the post-test.

The other shift we noted was from use of rules-based justification or claims without justification to use of justifications that were based on reasoning. For example, on the pre-test, several PSTs calculated, "$\frac{16}{10}\times\frac{10}{10}=\frac{160}{100}, \frac{160}{100}>\frac{134}{100},$" without any explanation. A typical rules-based responses was, "16 tenths is bigger than 134 hundredths. I decided that because when they have the same number of places, 160 is bigger than 134." On the post-test a PST compared 18-tenths and 172-hundredths, "If we rename the 8/10 left over into 10 times smaller pieces, there would be 80/100 pieces. Compared to 72/100 pieces, there are more with 80/100." Some PSTs referred to 10-by-10 grids in justifying why 18 tenths is the larger quantity: "180 hundredths vs. 172 hundredths. 18 columns with 10 boxes in each column. That gives you 180 boxes shaded, which is 180 hundredths because the columns are tenths and the boxes in the tenths are hundredths. 180-hundredths is more than 172-hundredths." There were also a few PSTs who made reference to everyday life contexts, such as currency: "I looked at it as 18 dimes, because a tenth of a dollar is a dime so if I had 18 dimes I have $1.80. If I had 172 hundredths that would be 172 pennies because a penny is a hundredth of a $1. So 1.80 is more than 1.72."

Conclusion

This study contributes to efforts to understand better how to support future elementary teachers in obtaining deep, connected knowledge of the mathematics they will teach. Prior to our decimals unit, future teachers struggled to understand the meanings of decimal numbers and to use those meanings to solve problems of equivalence and comparison. Whole group discussions about decimal renaming and introduction of representations such as 10-by-10 grids provided tools to help learners reason about the relative sizes of decimal quantities. Evidence from the post-test shows that PSTs began to visualize that 18 tenths makes more than a whole (10/10 + 8/10) and to explain why 8 tenths could be renamed as 80/100.

Throughout the course, we challenge our future teachers to analyze others' solutions and representations, both correct and incorrect. Pressing PSTs for justifications during whole group discussions encourages them to explain their reasoning and provide arguments convincing to others. Discussions that allow PSTs to examine and reflect on their own, often incorrect, procedural knowledge provide them with an opportunity to practice precise mathematical language they will use later in teaching.

The work described here helps to document changes in PST mathematical reasoning and representations, as they experience high-demand tasks and classroom implementation similar to those recommended by Stein et al. (2009) for classrooms of children. The results of this study suggest that these same strategies may support future teachers' development of mathematical knowledge they will need for teaching diverse groups of children.

References

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 29 (1), 14-22.

Common Core State Standards Initiative (CCSSI). (2010). *Common Core State Standards.* Washington, DC: The National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.coreStandards.org.

Conference Board of the Mathematical Sciences (2001). *The Mathematical Education of Teachers*, (Vol 11). Providence , RI: American Mathematical Society and Mathematical Association of America.

Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II*, (Vol 17). Providence , RI: American Mathematical Society and Mathematical Association of America.

Cramer, K. A., Monson, D. S., Wyberg, T., Leavitt, S., & Whitney, S. B. (2009). Models for initial decimal ideas. *Teaching Children Mathematics*, 16(2), 106-117.

D'Ambrosio, B. S. & Kastberg, S. E. (2012). Building understanding of decimal fractions. *Teaching Children Mathematics*, 18(9), 558-564.

Fuson, K. C. (1990). Conceptual structures for multidigit numbers: Implications for learning and teaching multidigit addition, subtraction, and place value. *Cognition and Instruction, 7*, 343–403.

Kamii, C. (1986). Place value: An explanation of its difficulty and educational implications for the primary grades. *Journal of Research in Childhood Education, 1*, 75–86.

Luo, F., Lo, J. J., & Leu, Y.C. (2011). Fundamental Fraction Knowledge of Preservice Elementary Teachers: A Cross-National Study in the United States and Taiwan. *School Science and Mathematics*, 111(4), 164-177.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Martinie, S. L. & Bay-Williams, J. M. (2003). Investigating students' conceptual understanding of decimal fractions using multiple representations. *Mathematics Teaching in the Middle School*, 8(5), 244-247.

Post, T. R., Harel, G., Behr, M. J., & Lesh, R. (1991). Intermediate teachers' knowledge of rational number concepts. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 177–198). Albany: State University of New York Press.

Putt, I. J. (1995). Preservice teachers' ordering of decimal numbers: When more is smaller and less is larger! *Focus on Learning Problems in Mathematics* 17(3), 1-15.

Rathouz, M. & Cengiz, N. (2012). Preservice teachers' understanding of decimal numbers and quantities. *Proceedings of Research Council on Mathematics Learning (RCML)*, Charlotte, NC.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education*, 20(1), 8-27.

Ross, S. H. (1990). Children's acquisition of place-value numeration concepts: The roles of cognitive development and instruction. *Focus on Learning Problems in Mathematics, 12*(1), 1–17.

Sowder, J. T., Philipp, R. A., Armstrong, B. E., & Schappelle, B. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction*. Albany: State University of New York Press.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education*, 4, 205-225.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (Second ed.). New York, NY: Teachers College Press.

Steinle, V. & Stacey, K. (2004). Persistence of decimal misconceptions and readiness to move to expertise. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 pp. 225-232.

Suh, J. M., Johnston, C., Jamieson, S., & Mills, M. (2008). Promoting decimal number sense and representational fluency. *Mathematics Teaching in the Middle School*, 14(1), 44-55.

Thanheiser, E. (2009). *Preservice elementary school teachers' conceptions of multidigit whole numbers.* Journal for Research in Mathematics Education, 40(3), 251-281.

Zazkis, R., & Khoury, H. A. (1993). Place value and rational number representations: Problem solving in the unfamiliar domain of non-decimals. *Focus on Learning Problems in Mathematics, 15*(1), 38–51.